

An equivalence principle for scalar forces

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The equivalence of inertial and gravitational masses is a defining feature of general relativity. Here, we clarify the status of the equivalence principle for interactions mediated by a universally coupled scalar, motivated partly by recent attempts to modify gravity at cosmological distances. Although a universal scalar-matter coupling is not mandatory, once postulated, it is stable against classical and quantum renormalizations in the matter sector. The coupling strength itself is subject to renormalization of course. The scalar equivalence principle is violated only for objects for which either the graviton self-interaction or the scalar self-interaction is important—the first applies to black holes, while the second type of violation is avoided if the scalar is Galilean-symmetric.

The defining feature of general relativity (GR) is undoubtedly the equivalence principle, namely the equivalence of inertial and gravitational masses. Phrased in a field theoretic language pioneered by Weinberg [1, 2], general relativity can be thought of as the unique (low energy) Lorentz-invariant theory of a massless spin-two field—the graviton. Consider the motion of an object, be it big as a galaxy or small as a proton, in a long-wavelength gravitational field—long compared to the object size. How the object responds to the gravitational field is captured by a coupling between the low energy graviton field and the object’s total physical energy-momentum. The equality of inertial and gravitational masses follows from their deriving from the same physical energy-momentum. We emphasize *physical*, meaning *renormalized*: the observed mass of an object typically has many contributions on top of its constituents’ rest masses, ranging from kinetic energies, to classical electromagnetic and gravitational potential energies, and to more genuinely quantum-mechanical corrections—which are in fact the leading source of mass for nucleons, for instance. Weinberg’s argument guarantees that each renormalization—classical or quantum-mechanical, gravitational or otherwise—of an object’s inertial mass is always accompanied by an *identical* renormalization of its gravitational mass. In other words, the universal coupling between the low energy graviton and objects is robust against renormalizations.

This renormalization property of gravitational interactions is what makes the equivalence principle of GR robust, or more precisely, stable under renormalization group: if a system’s constituents obey it individually, so does the system as a whole. Here we wish to establish the analogous statement for universal long-range forces mediated by scalar fields—that is, how robust is a universally coupled scalar? This question is far from academic, for most proposed theories that attempt to modify general relativity at cosmological distances, in one way or

another involve a scalar degree of freedom that couples to the trace of the matter stress-energy tensor. This includes the 5-dimensional DGP model [3, 4] and models of a massive/resonance graviton [5–7], not to mention classic scalar-tensor theories [8, 9]. We will see that the universal scalar coupling—a scalar equivalence principle—is robust against renormalizations in the matter sector. We will also see that violations arise only through nonlinearities in the graviton—the well known Nordtvedt effect [10]—and nonlinearities in the scalar which are crucial in recent theories for screening the scalar force on solar system scales [11].

Consider therefore a theory containing the gravitational field $g_{\mu\nu}$, matter, and a scalar ϕ that couples to all forms of matter energy-momentum with the same strength:

$$S = S_{\text{EH}}[g_{\mu\nu}] + S_{\text{m}}[g_{\mu\nu}, \psi] + \int d^4x \sqrt{-g} \{ \mathcal{L}_\phi[g_{\mu\nu}, \phi] + \phi T_{\text{m}}[g_{\mu\nu}, \psi] \} . \quad (1)$$

S_{EH} is the Einstein-Hilbert action; S_{m} and $T_{\text{m}}^{\mu\nu}$ are the action and stress-energy tensor for the matter degrees of freedom, collectively denoted by ψ . \mathcal{L}_ϕ encodes the dynamics of ϕ ; we are allowing for generic self-interactions. The coupling constant between ϕ and T_{m} is absorbed into the definition of ϕ itself (we will later discuss putting back an explicit coupling constant when addressing the renormalization of its *value*.) First of all, in comparison to GR, we already face a naturalness issue. The graviton’s coupling to matter is uniquely dictated by symmetry [1, 2]. For ϕ , there is no symmetry enforcing this particular form of the coupling— ϕ may couple to any local scalar quantity built out of ψ . We can however postulate the ϕT_{m} form of the coupling, and check whether it is stable under classical and quantum-mechanical renormalizations. That is, the best we can hope for is *technical* naturalness.

We can then ask, if ϕT_{m} is the correct coupling at some microscopic level, what is the coupling between a macroscopic or composite object and a long-wavelength ϕ field? The object is held together by internal forces, which may have any nature. For the moment, let us assume the gravitational contributions to the object’s total mass are

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negligible, though the object could still be gravitationally bound. Likewise, let's provisionally assume that ϕ 's contribution to the total mass and ϕ 's self interactions are also negligible. We can thus set $g_{\mu\nu} \simeq \eta_{\mu\nu}$, and in the point-particle limit for the object (that is, at lowest order in the multipole expansion), the scalar-object coupling is

$$\int d\tau Q \phi(x^\mu(\tau)) , \quad (2)$$

where Q is the object's scalar charge, which in the object's rest frame reads

$$Q \equiv \int d^3x T_m = \int d^3x (T_m^{00} - T_m^{ii}) . \quad (3)$$

The integral of T_m^{00} is just the total mass of the object, with no reference to how it is split into rest masses for the constituents, kinetic energies, and potential ones. The second term does not look as nice.

However, we can rewrite T_m^{ij} as

$$T_m^{ij} = \partial_k (x^i T_m^{kj}) - x^i \partial_k T_m^{kj} . \quad (4)$$

If we now integrate T_m^{ij} over space, the first piece yields zero for a localized system. The second piece, using stress-energy conservation, can be rewritten as a *time-derivative*:

$$\int d^3x T_m^{ij} = \partial_t \int d^3x x^i T_m^{0j} \quad (5)$$

(In fact by analogous arguments one can show that the integral of T_m^{ij} is a *second* time-derivative—this is one incarnation of the tensor virial theorem.) Therefore, for stationary systems the spatial integral of T_m^{ij} vanishes [19], and for virialized systems it averages to zero on time-scales larger than the system's dynamical time. Equivalently, for a low-frequency external ϕ field, this T_m^{ii} contribution to Q is negligible with respect to the T_m^{00} one. We are thus left with

$$Q \simeq M , \quad (6)$$

the total inertial mass of the object [20].

The above derivation shows that the equality of scalar charge and inertial mass is robust against classical renormalization. The same proof applies essentially unaltered to quantum mechanical contributions to Q , at the *non-perturbative* level. Consider for instance a proton, $|p\rangle$. If at the microscopic level ϕ couples as in eq. (1) to quarks and gluons, the coupling between our proton and a long wavelength ϕ will be eq. (2), with scalar charge

$$Q = \langle p | \int d^3x T_{\text{QCD}} | p \rangle , \quad (7)$$

where $T_{\text{QCD}}^{\mu\nu}(x)$ is the microscopic QCD stress-energy tensor operator, expressed in terms of quark and gluon fields. $T_{\text{QCD}}^{\mu\nu}(x)$ is conserved as an operator. We can thus run the same algebra as in the classical case, and end up with

$$Q = \langle p | \int d^3x T_{\text{QCD}}^{00} | p \rangle - \partial_t \langle p | \int d^3x x^i T_{\text{QCD}}^{0i} | p \rangle . \quad (8)$$

The first term is the physical mass of the proton—by definition. The second term vanishes, because the proton is a non-perturbative stationary state of the QCD Hamiltonian. We thus have

$$Q = M , \quad (9)$$

like in the classical case.

Given its dryness, our proof deserves some comments. The crucial ingredient we are relying on is stress-energy conservation for matter alone. As is clear from our action (1), we are calling ‘matter’ everything but the gravitational field and ϕ itself. We have thus demonstrated the robustness of the scalar equivalence principle against renormalizations in the matter sector only. What is exactly conserved is, of course, the total stress-energy (pseudo-)tensor $t_{\mu\nu}$ for matter, gravity, and ϕ . This means that our result does not apply to systems where gravity or ϕ gives sizable contributions to the total mass, like a black hole. Indeed, because of the no-hair theorem a black hole cannot couple to a long-wavelength scalar field, thus violating $Q \simeq M$ —the Nordtvedt effect [10, 13]. Note however that our result does apply to gravitationally or ‘scalar-ly’ bound systems with negligible gravitational and scalar self-energy—in such a case $T_m \simeq t^\mu{}_\mu$, and in our proof we could have just used $t_{\mu\nu}$, which is exactly conserved. More importantly, our proof neglects contributions to the scalar charge from ϕ 's self-interactions, which as we mentioned are crucial for screening the scalar force in the solar system. Such self-interactions effectively renormalize the monopole coupling (3) of a localized object to a long-wavelength ϕ by an amount

$$\Delta Q = \int d^3x \frac{\partial \mathcal{L}_\phi}{\partial \phi} [\phi_{\text{obj}}] , \quad (10)$$

where ϕ_{obj} is the ϕ field dressing the object. Notice that here the derivative w.r.t. ϕ is an ordinary one—not a functional one [11]. This is because we want to isolate the monopole coupling, which involves the zero-momentum limit for ϕ . For generic \mathcal{L}_ϕ , this contribution can yield violations of the equivalence principle of order one, such as in chameleon [14] or symmetron [15] screening. However there exists a class of observationally viable scalar self-couplings—those associated with Galilean invariance [16], used in Vainshtein screening [17]—that do not renormalize the scalar charge [11]. For these interactions the equivalence principle is preserved as long as the gravitational and scalar binding energies are negligible.

Our simple derivation sheds light on what might otherwise appear to be miraculous cancellations in computations of quantum corrections to the scalar coupling. The robustness of the universal coupling has been demonstrated by Fujii [18] in the context of a scalar coupled to QED. Let's consider as another example a matter sector with a set of interacting scalars ψ_a with Lagrangian

$$\mathcal{L}_m = \sum_a \frac{1}{2} [(\partial \psi_a)^2 - m_a^2 \psi_a^2] - \sum_{a,b} \lambda_{ab} \psi_a^2 \psi_b^2 . \quad (11)$$

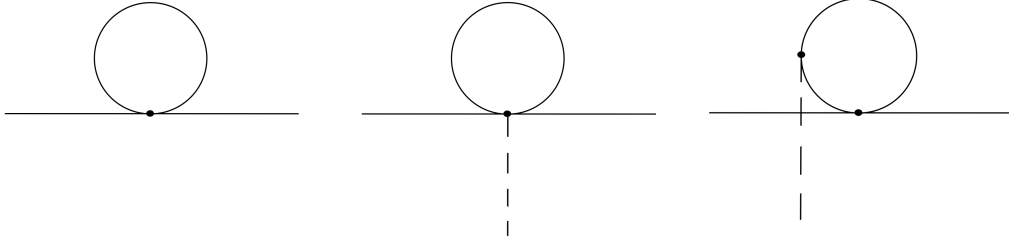


FIG. 1: One-loop contributions to the quadratic effective action for ψ (solid lines) in the presence of an external ϕ (dashed line).

For simplicity we are postulating a symmetry under $\psi_a \rightarrow -\psi_a$ for each particle species, so that quantum corrections do not generate kinetic and mass-mixings between different species. Let's couple our ϕ to the ψ 's stress-energy tensor, like in eq. (1). We have

$$T_m = \sum_a \frac{1}{2} [-(d-2)(\partial\psi_a)^2 + d m_a^2 \psi_a^2] + d \sum_{a,b} \lambda_{ab} \psi_a^2 \psi_b^2, \quad (12)$$

where d is the spacetime dimensionality—we will use dimensional regularization for the UV divergences. From fig. 1, we can compute the 1-loop contribution to the quadratic effective action for the ψ 's, in the presence of a long-wavelength external ϕ . In fact the computation is made simpler by noting that combining the matter Lagrangian with the interaction with ϕ we get the same structure as in eq. (11), but with ϕ -dependent coefficients in front of each term. For a very long wavelength ϕ , we can treat such coefficients as constant. Upon redefining $\psi_a \rightarrow [1 + \frac{1}{2}(d-2)\phi]\psi'_a$ to re-gain canonical normalization, we get

$$\mathcal{L}_m + \phi T_m \rightarrow \sum_a \frac{1}{2} [(\partial\psi'_a)^2 - \hat{m}_a^2(\phi) \psi'^2_a] - \sum_{a,b} \hat{\lambda}_{ab}(\phi) \psi'^2_a \psi'^2_b, \quad (13)$$

where

$$\hat{m}_a^2(\phi) \equiv m_a^2[1-2\phi], \quad \hat{\lambda}_{ab}(\phi) \equiv \lambda_{ab}[1+(d-4)\phi], \quad (14)$$

and we kept the linear order in ϕ only. It is now simple to retrieve quantum corrections to how ϕ couples to the ψ 's—in the zero momentum limit for ϕ —from loop diagrams with no external ϕ -lines. For instance, now the leftmost diagram of fig. 1 is enough to produce the 1-loop effective action at quadratic order in the ψ 's:

$$\Delta\mathcal{L} = -\frac{1}{2} \sum_a \frac{\Delta m_a^2}{m_a^2} \hat{m}_a^2(\phi) \psi'^2_a, \quad (15)$$

where

$$\Delta m_a^2 \propto \sum_b (2\delta_{a \neq b} + 12\delta_{ab}) \lambda_{ab} \left[\frac{1}{d-4} + \log(m_b/\mu) \right] m_b^2. \quad (16)$$

That is, for each particle species, the coupling to ϕ gets renormalized precisely by the same multiplicative factor as the inertial mass, as predicted. Equivalently, undoing the field redefinition and expressing everything in terms of the original ψ , at quadratic order in ψ the coupling with ϕ is still of the ϕT_m form:

$$\mathcal{L}_m + \phi T_m \rightarrow \mathcal{L} + \Delta\mathcal{L}_m + \phi (T_m + \Delta T_m), \quad (17)$$

where $\Delta T_m^{\mu\nu}$ is the correction to $T_m^{\mu\nu}$ associated with $\Delta\mathcal{L}_m$, i.e.

$$\Delta T_m^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \Delta\mathcal{L}_m}{\delta g_{\mu\nu}}. \quad (18)$$

Notice that this relies crucially on the mass-term quantum correction (15)'s having the same universal ϕ -dependence as the tree-level mass term in (13)—proportional to $(1-2\phi)$. If the ϕ -dependence were different from the tree-level one, and species-dependent, it would lead to two different kinds of particles falling at different rates. The ‘miraculous cancellations’ alluded to above, are here embodied by the equality

$$\hat{\lambda}_{ab}(\phi) \left[\frac{1}{d-4} + \log(\hat{m}_b(\phi)/\mu) \right] = \lambda_{ab} \left[\frac{1}{d-4} + \log(m_b/\mu) \right], \quad (19)$$

which is valid at first order in ϕ and which was crucial to obtain the universal structure (15). Given our general arguments above, we expect eq. (17) to hold at ψ^4 order as well.

This example thus illustrates and confirms our general result for a universal scalar-matter coupling. The universality can be violated only when gravitational or scalar self-interactions are important, with the latter possibility precluded in theories with Galilean symmetry. In this regard, scalar forces are capable of obeying an equivalence principle, though one not as strong and inevitable as that for the graviton.

We close with a few final remarks. First, our general analysis and our one-loop example show that starting with a ϕT_m coupling, upon renormalization the coupling between ϕ and matter will remain precisely ϕT_m , if T_m is expressed in terms of the physical, renormalized masses and couplings. The overall coefficient in front does not get renormalized. However the Lagrangian describing the

dynamics of ϕ does get renormalized—for instance matter loops with two ϕ -external legs yield wave-function renormalizations for ϕ . So, in the end, the universal coupling between matter and ϕ -quanta—which we get by going to canonical normalization for ϕ —does receive (universal) quantum corrections. More explicitly, a canonically normalized ϕ couples to matter as $\alpha\phi T_m$, and the value of α is subject to renormalization. For instance, the value $\alpha = 1/\sqrt{6}$ that defines $f(R)$ is not protected. Perhaps more importantly, matter loops with an external ϕ and an external graviton will generically generate a kinetic mixing between the two fields. Demixing them from each other will not affect the universality of the scalar coupling to matter though, for the graviton is itself universally coupled.

Second, our proof generalizes straightforwardly to a symmetric tensor field coupled to the matter stress-energy tensor, $\mathcal{L} \supset X_{\mu\nu} T_m^{\mu\nu}$, of which our scalar coupling is just a special case with $X_{\mu\nu} = \phi \eta_{\mu\nu}$. Indeed the point-particle coupling (2) generalizes to

$$\int d\tau Q^{\mu\nu} X_{\mu\nu}(x^\mu(\tau)) , \quad (20)$$

where the ‘tensor charge’ $Q^{\mu\nu}$ is defined as

$$Q^{\mu\nu} \equiv \int d^3x T_m^{\mu\nu} , \quad (21)$$

in the object’s rest frame. T_m^{ij} integrates to zero because

of the same reasons (and under the same assumptions) as above; T_m^{0i} integrates to the total momentum, which also vanishes in the object’s rest frame. We thus have that in the rest frame the only non-vanishing entry of $Q^{\mu\nu}$ is $Q^{00} = M$ —the total inertial mass of the object. Going to a generic frame we thus get that the coupling (20) reduces to

$$M \int d\tau u^\mu u^\nu X_{\mu\nu}(x^\mu(\tau)) , \quad (22)$$

where u^μ is the objects four-velocity. This is precisely how the gravitational field couples to an object in the point-particle approximation. Clearly the same proof applies in the quantum-mechanical case, with the same modifications as above.

Finally, in the quantum-mechanical case we have been neglecting loop contributions with ϕ and graviton internal lines. These are suppressed by inverse powers of the Planck mass (assuming the scalar couples to matter with gravitational strength), and can be safely neglected as long as matter self-interactions are much stronger than gravity.

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